

# Thermal Model Description of Collisions of Small Nuclei

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*Dedicated to the memory of Helmut Oeschler*

**Abstract** The dependence of particle production on the size of the colliding nuclei is analysed in terms of the thermal model using the canonical ensemble. The concept of strangeness correlation in clusters of sub-volume  $V_c$  is used to account for the suppression of strangeness. A systematic analysis is presented of the predictions of the thermal model for particle production in collisions of small nuclei. The pattern of the maxima of strange-particles-to-pion ratios as a function of beam energy is quite special, as they do not occur at the same beam energy and are sensitive to system size. In particular, the  $\Lambda/\pi^+$  ratio shows a clear maximum even for small systems while the maximum in the  $K^+/\pi^+$  ratio is less pronounced in small systems.

## 1 Introduction

A substantial experimental effort is presently under way to study not only heavy- but also light-ion collisions. This is being motivated by the puzzling results, obtained in Pb-Pb and Au-Au collisions, for the non-monotonic behavior of the

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$K^+/\pi^+$  ratio, and other particle ratios which have been conjectured as being due to a phase change in nuclear matter [1].

A consistent description of particle production in heavy-ion collisions, up to LHC energies, has emerged during the past two decades using a thermal-statistical model (referred to simply as thermal model in the remainder of this paper). It is based on the creation and subsequent decay of hadronic resonances produced in chemical equilibrium at a unique temperature and baryo-chemical potential. According to this picture the bulk of hadronic resonances made up of the light flavor (u,d and s) quarks are produced in chemical equilibrium.

Indeed, some particle ratios exhibit very interesting features as a function of beam energy which deserve attention: (i) a maximum in the  $K^+/\pi^+$  ratio, (ii) a maximum in the  $\Lambda/\pi$  ratio, (iii) no maximum in the  $K^-/\pi^-$  ratio. These three features occur at a centre-of-mass energy of around 10 GeV [2,3,4]. The maxima happen in an energy regime where the largest net baryon density occurs [5,6] and a transition from a baryon-dominated freeze out to a meson dominated one takes place [4]. An alternative interpretation is that these maxima reflect a phase change to a deconfined state of matter [1].

The maxima mark a distinction between heavy-ion collisions and p-p collisions as they have not been observed in the latter. This shows a clear difference between the two systems which is worthy of further investigation.

It is the purpose of the present paper to investigate the transition from a small system like a p-p collision to a large system like a Pb-Pb or Au-Au collision and to follow explicitly the genesis of the maxima in certain particle ratios. This is relevant for the interpretation of the data coming out of the BES [7] and NA61 [8] experiments. These experiments are spearheading this effort at the moment, in the near future additional results will be obtained at the NICA collider and at the FAIR facility.

## 2 The model

A relativistic heavy-ion collision goes through several stages. At one of the later stages, the system is dominated by hadronic resonances. The identifying feature of the thermal model is that all the resonances as listed by the Particle Data Group [9] are assumed to be in thermal and chemical equilibrium. This assumption drastically reduces the number of free parameters and thus this stage is determined by just a few thermodynamic variables namely, the chemical freeze-out temperature  $T$ , the various chemical potentials  $\mu$  determined by the conserved quantum numbers and by the volume  $V$  of the system. It has been shown that this description is also the correct one [10,11,12] for a scaling expansion as first discussed by Bjorken [13].

In general, if the number of particles carrying quantum numbers related to a conservation law is small, then the grand-canonical description no longer holds. In such a case, conservation of charges has to be implemented exactly by using the canonical ensemble [14,15,16,17]. We start by presenting a brief reminder of the general concepts of the thermal model.

## 2.1 Grand Canonical Ensemble

In the grand-canonical ensemble, the volume  $V$ , temperature  $T$  and the chemical potentials  $\boldsymbol{\mu}$  determine the partition function  $Z(T, V, \boldsymbol{\mu})$ . In the hadronic fireball of non-interacting hadrons,  $\ln Z$  is the sum of the contributions of all  $i$ -particle species given by

$$\frac{1}{V} \ln Z(T, V, \boldsymbol{\mu}) = \sum_i Z_i^1(T, \boldsymbol{\mu}), \quad (1)$$

where  $\boldsymbol{\mu} = (\mu_B, \mu_S, \mu_Q)$  are the chemical potentials related to the conservation of baryon number, strangeness and electric charge, respectively.

The partition function contains all information needed to obtain the number density  $n_i$  of particle species  $i$ . Introducing the particle's specific chemical potential  $\mu_i$ , one gets

$$n_i(T, \boldsymbol{\mu}) = \frac{1}{V} \left. \frac{\partial(T \ln Z)}{\partial \mu_i} \right|_{\mu_i=0}. \quad (2)$$

Any resonance that decays into species  $i$  contributes to the yields eventually measured. Therefore, the contributions from all heavier hadrons  $j$  that decay to hadron  $i$  with the branching fraction  $\Gamma_{j \rightarrow i}$  are given by:

$$n_i^{\text{decay}} = \sum_j \Gamma_{j \rightarrow i} n_j. \quad (3)$$

Consequently, the final yield  $N_i$  of particle species  $i$  is the sum of the thermally produced particles and the decay products of resonances,

$$N_i = (n_i + n_i^{\text{decay}}) V. \quad (4)$$

From Eqs. (2-4) it is clear that in the grand-canonical ensemble the particle yields are determined by the volume of the fireball, its temperature and the chemical potentials.

## 2.2 Canonical ensemble

If the number of particles is small, then the grand-canonical description no longer holds. In such a case conservation laws have to be implemented exactly. Here, we refer only to strangeness conservation and consider charge and baryon number conservation to be fulfilled on the average in the grand canonical ensemble because the number of charged particles and baryons is much larger than that of strange particles [14]. The density of strange particle  $i$  carrying strangeness  $s$  can be obtained from (see [14] for further details),

$$n_i^C = \frac{Z_i^1}{Z_{S=0}^C} \sum_{k=-\infty}^{\infty} \sum_{p=-\infty}^{\infty} a_3^p a_2^k a_1^{-2k-3p-s} I_k(x_2) I_p(x_3) I_{-2k-3p-s}(x_1), \quad (5)$$

where  $Z_{S=0}^C$  is the canonical partition function

$$Z_{S=0}^C = e^{S_0} \sum_{k=-\infty}^{\infty} \sum_{p=-\infty}^{\infty} a_3^p a_2^k a_1^{-2k-3p} I_k(x_2) I_p(x_3) I_{-2k-3p}(x_1), \quad (6)$$

and  $Z_i^1$  is the one-particle partition function (Eq. 6) calculated for  $\mu_S = 0$  in the Boltzmann approximation. The arguments of the Bessel functions  $I_s(x)$  and the parameters  $a_i$  are introduced as,

$$a_s = \sqrt{S_s/S_{-s}} \quad , \quad x_s = 2V\sqrt{S_s S_{-s}}, \quad (7)$$

where  $S_s$  is the sum of all  $Z_k^1$  for particle species  $k$  carrying strangeness  $s$ .

In the limit where  $x_n < 1$  (for  $n = 1, 2$  and  $3$ ) the density of strange particles carrying strangeness  $s$  is well approximated by [14]

$$n_i^C \simeq n_i \frac{I_s(x_1)}{I_0(x_1)}. \quad (8)$$

From these equations it is clear that in the canonical ensemble the strange particle density depends explicitly on the volume through the arguments of the Bessel functions. This volume might be different from the overall volume  $V$  and is denoted as  $V_c$  [16, 17, 18].

In this case there are two volume parameters: the overall volume of the system  $V$ , which determines the particle yields at fixed density and the strangeness correlation volume  $V_c$ , which describes the range of strangeness conservation. If this volume is small, it reduces the densities of strange particles.

### 3 Origin of the maxima

It has been observed that the baryon chemical potential decreases with increasing beam energy while the temperature increases quickly and reaches a plateau. Following the rapid rise of the temperature at low beam energies, the  $\Lambda/\pi^+$  and  $K^+/\pi^+$  also increase rapidly. This halts when the temperature reaches its limiting value. Simultaneously the baryon chemical potential keeps on decreasing. Consequently, the  $\Lambda/\pi$  and  $K^+/\pi^+$  ratios follow this decrease due to strangeness conservation as  $K^+$  is produced in associated production together with a  $\Lambda$ . The two effects combined lead to maxima in both cases. For very high energies, the baryochemical potential no longer plays a role ( $\mu_B \approx 0$ ) and, since the temperature remains constant, these ratios no longer vary [4].

In order to analyse the strangeness content in a heavy ion collision we make use of the Wroblewski factor [19] which is defined as

$$\lambda_s = \frac{2 \langle s\bar{s} \rangle}{\langle u\bar{u} \rangle + \langle d\bar{d} \rangle}.$$

This factor is determined from the quark content of the hadronic resonances, namely, by the number of newly created strange – anti-strange ( $\langle s\bar{s} \rangle$ ) and the non-strange ( $\langle u\bar{u} \rangle$  and  $\langle d\bar{d} \rangle$ ) quarks *before* their strong decays.

Its limiting values are obvious:  $\lambda_s = 1$ , if all quark pairs are equally abundantly produced, i.e. flavor SU(3) symmetry and  $\lambda_s = 0$ , if no strange quark pairs are present in the final state. This factor has been calculated in the thermal model using the THERMUS [21] code by examining the quark content of hadronic resonances. Due to its definition, contributions from heavy flavors like charm are explicitly excluded. The relative strangeness production in heavy-ion collisions along the chemical freeze-out line shows a maximum. This is illustrated by the

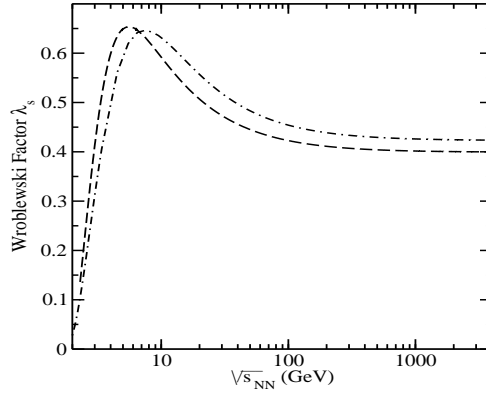


Fig. 1: The Wroblewski factor  $\lambda_s$  as a function of beam energy calculated along the chemical freeze-out curve. The dashed-dotted line is calculated along the freeze-out curve obtained in [3] while the dashed line uses the parametrization given in [20].

Wroblewski factor in Fig. 1. The functional dependence of  $T$  and  $\mu_B$  on  $\sqrt{s_{NN}}$  as in [3] was used. For comparison we also show the freeze-out parameterization recently introduced in [20], which has a lower limiting temperature than the one given in [3].

In an earlier publication, we have already discussed how the maximum in  $\lambda_s$ , seen in Fig. 1, occurs [14]. It turns out that the shape of the Wroblewski factor tracks the energy dependence of the  $K^+/\pi^+$  ratio.

To show this in more detail we present as an example in Fig. 2 contour lines where the  $K^+/\pi^+$  and the  $\Lambda/\pi^+$  ratios remain constant in the  $T - \mu_B$  plane. It should be noted that the maxima of these ratios do not occur in the same position, which remains to be confirmed experimentally. It is also worth noting that in these cases the maxima are not on, but slightly above the freeze-out curve.

#### 4 Particle ratios for small systems

To consider the case of the collisions of smaller nuclei we take into account the strangeness suppression using the concept of strangeness correlation in clusters of a sub-volume  $V_c \leq V$  [16, 17, 22].

A particle with strangeness quantum number  $s$  can appear anywhere in the volume  $V$  but it has to be accompanied by another particles carrying strangeness  $-s$  to conserve strangeness in the correlation volume  $V_c$ . Assuming spherical geometry, the volume  $V_c$  is parameterized by the radius  $R_c$  which is a free parameter that defines the range of local strangeness equilibrium.

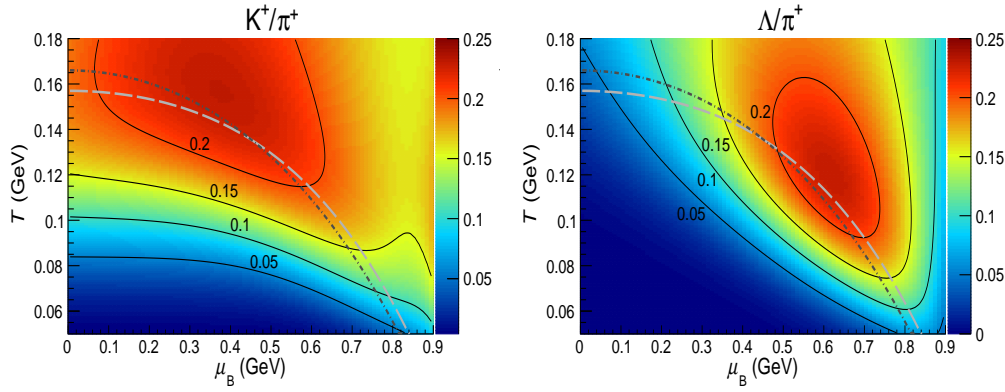


Fig. 2: Values of the  $K^+/\pi^+$  (left-hand pane) and the  $\Lambda/\pi^+$  (right-hand pane) ratios in the  $T - \mu_B$  plane. Lines of constant values are indicated. The dashed-dotted line is the freeze-out curve obtained in [3] while the dashed line uses the parameterization given in [20]. Note that the maxima do not occur in the same position.

In the following we show the trends of various particle ratios as a function of  $\sqrt{s_{NN}}$ . The dependence of  $T$  and  $\mu_B$  on the beam energy is taken from heavy-ion collisions [3]. For p-p collisions slightly different parameters would be more suited [23]. Therefore, the calculations illustrate the general trend, as we have ignored the variations of the parameters with system size.

We focus on the system-size dependence of the thermal parameters with particular emphasis on the change in the strangeness correlation radius  $R_c$ . The radius parameters of the volume  $V$ ,  $R = 10$  fm (which is the value for central Pb-Pb collisions) and  $\gamma_S = 1$  are kept fixed. The freeze-out values of  $T$  and  $\mu_B$  will vary with the system size [22], however this has not been taken into account in the present work which therefore gives only a qualitative description of the effect.

The smaller system size is described by decreasing the value of the correlation radius  $R_c$ . This ensures that strangeness conservation is exact in  $R_c$ , and hence, strangeness production is more suppressed with decreasing  $R_c$ .

In Fig. 3 we show the energy and system size dependence of different particle ratios calculated along the chemical freeze-out line. A maximum is seen in the  $K^+/\pi^+$  ratio which gradually disappears when the correlation radius decreases. A different effect is seen in  $\Lambda/\pi^\pm$  ratio. Here, the gradual decrease of the maximum is also seen but, contrary to the  $K^+/\pi^+$  ratio, it does not disappear and is still present even for small radii. This behavior can be tested experimentally and, if confirmed, will give support the hadronic scenario presented here.

In the left-hand panel of Fig. 3 it is also seen that for different particle ratios the maxima gradually become less pronounced as the size of the system decreases. Also, the maximum shifts, for smaller systems, towards higher  $\sqrt{s_{NN}}$ . For pp collisions which correspond to a  $R_c$  of about 1.5 fm [22], they will hardly be observed,

except for  $\Lambda/\pi$  ratio. As noted before the maxima happen at different beam energies.

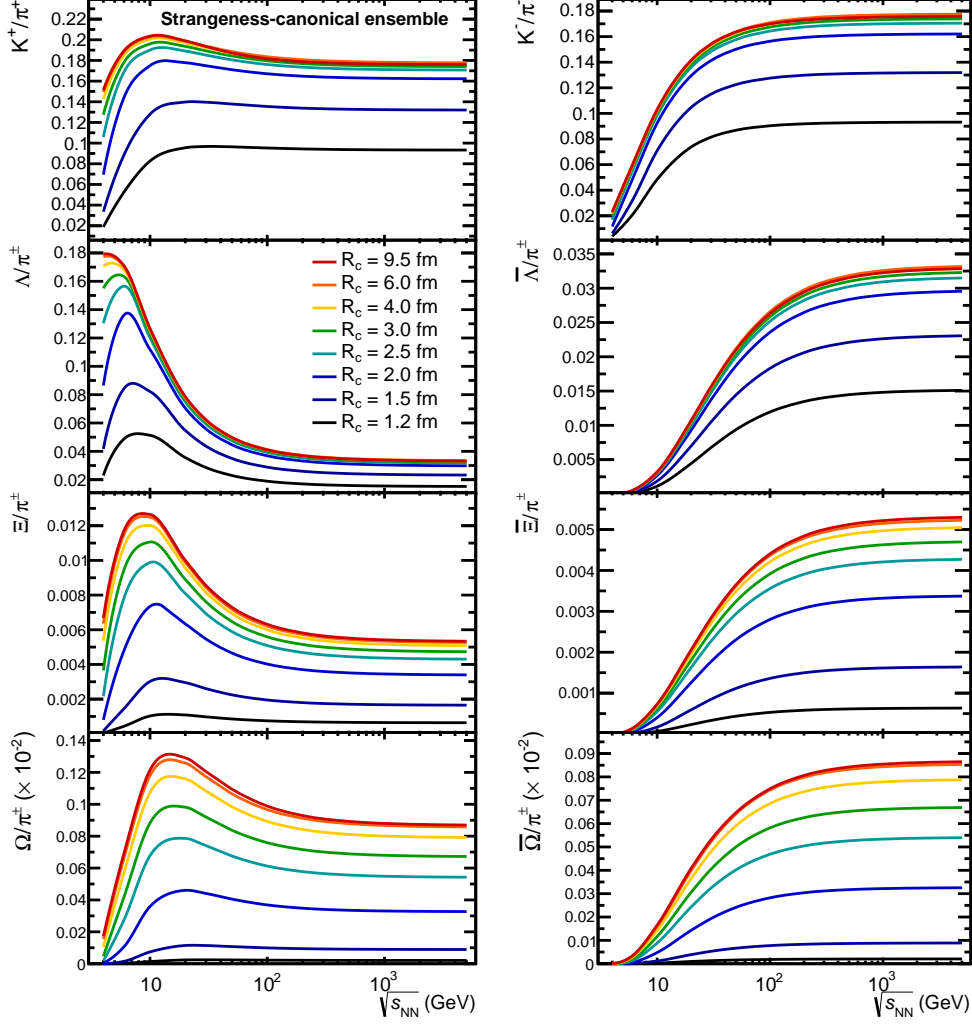


Fig. 3: Behavior of particle ratios as a function of the invariant beam energy for various strangeness correlation radii  $R_c$ , calculated using the thermal model formulated in a canonical ensemble [21]. The  $K^+/\pi^+$ ,  $\Lambda/\pi^\pm$ ,  $\Xi/\pi^\pm$  and  $\Omega/\pi^\pm$  ratios are shown in the left-hand panel while the corresponding antiparticles are shown in the right-hand panel. Note that the  $\Lambda/\pi^\pm$  ratio is the only ratio where the maximum does not disappear as the system size is reduced.

The corresponding ratios for antiparticles are shown in the right-hand panel of Fig. 3. As is to be expected in the thermal model, no maxima are present because

the baryon chemical potential  $\mu_B$  enters with the opposite sign and the ratios increase smoothly with increasing beam energies until they reach a constant value corresponding to the limiting hadronic temperature.

The ratios involving multi-strange baryons are also shown in Fig. 3. It is to be noted here that the maxima occur at a higher beam energy than for the  $K^+/\pi^+$  and the  $\Lambda/\pi$  ratios. *The maxima are caused by an interplay of different mass thresholds, a decreasing  $\mu_B$  and the saturation of  $T$ .* The maxima gradually disappear as the size of the system is reduced. These are the main results of the present paper.

It must be emphasized that the results presented here are of a qualitative nature. In particular there could be changes due to variations with the system size of the temperature and the baryon chemical potential. In addition the strangeness equilibration volume  $V_c$  could be energy dependent and system-size dependent.

## 5 Conclusions

The thermal model describes the presence of maxima in the  $K^+/\pi^+$  and the  $\Lambda/\pi^\pm$  ratios at a beam energy of  $\sqrt{s_{NN}} \approx 10$  GeV. In this paper we have described what could possibly happen with different strange particles and pion yields in collisions of smaller systems due to constraints imposed by exact strangeness conservation. To this end, use was made of a correlation volume to account for the strangeness suppression effect. We have shown that, in general, the characteristic feature of such ratios is a non-monotonic excitation function with well identified maxima for the ratios involving strange particles. The ratios with the anti-strange particle yields exhibit a monotonic increase and saturation with energy. A decrease in the maxima was noted and for certain ratios of particle yields the maxima completely disappear but not for all. In particular, the  $\Lambda/\pi^+$  ratio still shows a clear maximum even for small systems. The pattern of these maxima is also quite special, they are not always at the same beam energy.

If all ratios are following the trend given here, it is a strong argument that the properties of the strange particle excitation functions, and their system size dependence, are governed by the hadronic phase of the collisions constrained by an exact strangeness conservation implemented in a canonical ensemble.

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